Math Virtual Learning

Algebra IIB

Using an Exponential Equation to Find Half-Life

April 21, 2020

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Lesson: April 21, 2020

Objective/Learning Target: Students will use exponential equations find the half-life of isotopes

Let's Get Started:

Half-life is the time it takes for the radioactivity of a specified isotope to fall to half its original value. Why is this important? It is used frequently in science. Carbon dating is an application of half-life. It helps to determine the age of organic objects. In the medical field it is used in radiotherapy to reduce the size of cancerous tumors. Watch this video to get an introduction into how half-life is calculated:

Introduction to half-life

Exponential Half-Life Formula

We need to use a formula to get a more precise answer. This is one form of the half-life formula:

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

A_0: The original amount of the isotope
h: half-life
t: time
*** the half-life and time have to be in the same unit of time!

Example 1:

An alien radioactive isotope has a half-life of 238 years. If you start with a sample of 8 kg, how much will be left in 100 years?

Step 1: Define the variables:

$h=238$

$A_0=8$

$t=100$

Step 2: Fill in the formula:

$A=A_0(½)^{t/h}$

$A=8(½)^{100/238}$

Step 3: Simplify

$A=5.98$ kg

Your Turn!

You discovered a new radioactive isotope and named it boogonium (don't ask). It's half life is **1.23** years. If you start with a sample of **45** grams, how much will be left in **6.7** years?

Answer to Practice

You discovered a new radioactive isotope and named it boogonium (don't ask). It's half life is 1.23 years. If you start with a sample of 45 grams, how much will be left in 6.7 years?

Step 1: Define the variables:

$h=1.23$

$A_0=45$

$t=6.7$

Step 2: Fill in the formula:

$A=A_0(½)^{t/h}$

$A=45(½)^{6.7/1.23}$

Step 3: Simplify

$A=1.03$ g

Logarithmic Half-Life Formula

We need to rearrange the formula into a logarithm to solve for time or half-life.

$$A = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}} \rightarrow \frac{A}{A_0} = \left(\frac{1}{2}\right)^{\frac{t}{h}} \rightarrow \ln\left(\frac{A}{A_0}\right) = \ln\left(\frac{1}{2}\right)^{\frac{t}{h}} \rightarrow \ln\left(\frac{A}{A_0}\right) = \frac{t}{h}\ln\left(\frac{1}{2}\right)$$

Using the natural log formula, rearrange it to solve for h and t.

Rearranged Formulas:

Solved for half-life:

$$\ln\left(\frac{A}{A_0}\right) = \frac{t}{h}\ln\left(\frac{1}{2}\right) \rightarrow \quad h\ln\left(\frac{A}{A_0}\right) = t\ln\left(\frac{1}{2}\right) \rightarrow h = t \cdot \frac{\ln(0.5)}{\ln\left(\frac{A}{A_0}\right)}$$

Solved for time:

$$\ln\left(\frac{A}{A_0}\right) = \frac{t}{h}\ln\left(\frac{1}{2}\right) \rightarrow \quad h\ln\left(\frac{A}{A_0}\right) = t\ln\left(\frac{1}{2}\right) \rightarrow t = h \cdot \frac{\ln\left(\frac{A}{A_0}\right)}{\ln(0.5)}$$

Example 2:

You are observing a mystery radioactive isotope. At 4 pm, there are 2.5 grams and at 9 pm, there are 1.7 grams. What's the half-life?

Step 1: Define the variables:

A_0=2.5
A=1.7
t=5
(9PM-4PM)

Step 2: Fill in the formula:

$$h = t \cdot \frac{\ln(0.5)}{\ln\left(\frac{A}{A_0}\right)}$$

$$h = 5\left(\frac{\ln(0.5)}{\ln\left(\frac{1.7}{2.5}\right)}\right)$$

Step 3: Simplify

h=8.99 hours

Independent Practice

Do the attached worksheet.
The answers are on the next slide.

Half-Life Worksheet

Hint for the worksheet: If the problem gives you the half-life and then asks how much is left after 5 half-lifes, that means that t=5h. Look what happens to the exponent:

$$\frac{t}{h} = \frac{5h}{h} = 5$$

Answers to Independent Practice

1. 0.25 mg
2. 5 mg; 2.5 mg; 1.25 mg
3. 800 mg
4. 0.03 g
5. 0.0625 g
6. 3 mg; 0.1875 mg
7. 5.16 g
8. Approximately 9; 0.625 mg